Unit 28: Further Mathematics for Technicians

NQF Level 3: BTEC National

Guided learning hours: 60

Unit abstract

Mathematics is an essential tool for any electrical or mechanical engineering technician. This unit has been designed to further enhance learners' knowledge of mathematical principles, particularly for those considering progressing to a higher education qualification in engineering. With this in mind, the learning outcomes offer greater emphasis to the engineering application of mathematics. For example, learners could use an integral calculus method to obtain the root mean square (RMS) value of a sine wave over a half cycle.

The first learning outcome will extend learners' knowledge of graph plotting and will develop the technique of using a graph to solve (find the roots of), for example, a quadratic equation.

Learning outcome 2 involves the use of both arithmetic and geometric progressions for the solution of practical problems. The concept of complex numbers, an essential tool for electrical engineers considering, is also introduced.

Learning outcome 3 considers the parameters of trigonometrical graphs and the resultant wave when two are combined. The use of mathematical formulae in the latter half of this learning outcome enables a mathematical approach to wave combination to be considered.

Finally, in learning outcome 4, calculus techniques are further developed and used to show their application in engineering.

Learning outcomes

On completion of this unit a learner should:

1 Be able to use advanced graphical techniques

2 Be able to apply algebraic techniques

3 Be able to understand how to manipulate trigonometric expressions and apply trigonometric techniques

4 Be able to apply calculus.

Unit content

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1 Be able to use advanced graphical techniques

Advanced graphical techniques: graphical solution eg of a pair of simultaneous equations with two unknowns, to find the real roots of a quadratic equation, for the intersection of a linear and a quadratic equation, non-linear laws such as ($y = ax^2 + b,\ y = a + \frac{b}{x}$), by the use of logarithms to reduce laws of type $y = ax^n$ to straight line form, of a cubic equation such as $2x^3 - 7x^2 + 3x + 8 = 0$, recording, evaluating and plotting eg manual, computerised

2 Be able to apply algebraic techniques

Arithmetic progression (AP): first term (a), common difference (d), nth term eg $a + (n-1)d$; arithmetic series eg sum to n terms, $S_n = \frac{n}{2}\{2a + (n-1)d\}$

Geometric progression (GP): first term (a), common ratio (r), nth term eg $a\,r^{n-1}$; geometric series eg sum to n terms, $S_n = \frac{a(r^n - 1)}{r - 1}$, sum to infinity $S_\infty = \frac{a}{1 - r}$; solution of practical problems eg compound interest, range of speeds on a drilling machine

Complex numbers: addition, subtraction, multiplication of a complex number in Cartesian form, vector representation of complex numbers, modulus and argument, polar representation of complex numbers, multiplication and division of complex numbers in polar form, polar to Cartesian form and vice versa, use of calculator

Statistical techniques: review of measure of central tendency, mean, standard deviation for ungrouped and grouped data (equal intervals only), variance

3 Be able to understand how to manipulate trigonometric expressions and apply trigonometric techniques

Trigonometrical graphs: amplitude, period and frequency, graph sketching eg sin x, 2 sin x, ½ sin x, sin 2x, sin ½ x for values of x between 0 and 360°; phase angle, phase difference; combination of two waves of the same frequency

Trigonometrical formulae and equations: the compound angle formulae for the addition of sine and cosine functions eg sin (A ± B); expansion of R sin (wt + α) in the form a cos wt + b sin wt and vice versa

4 **Be able to apply calculus**

Differentiation: review of standard derivatives, differentiation of a sum, function of a function, product and quotient rules, numerical values of differential coefficients, second derivatives, turning points (maximum and minimum) eg volume of a rectangular box

Integration: review of standard integrals, indefinite integrals, definite integrals eg area under a curve, mean and RMS values; numerical eg trapezoidal, mid-ordinate and Simpson's rule

Grading grid

In order to pass this unit, the evidence that the learner presents for assessment needs to demonstrate that they can meet all of the learning outcomes for the unit. The criteria for a pass grade describes the level of achievement required to pass this unit.

Grading criteria					
To achieve a pass grade the evidence must show that the learner is able to:		**To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:**		**To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:**	
P1	use a graphical technique to solve a pair of simultaneous linear equations	M1	use the laws of logarithms to reduce an engineering law of the type $y = ax^n$ to straight line form, then using logarithmic graph paper, plot the graph and obtain the values for the constants *a* and *n*	D1	using a graphical technique determine the single wave resulting from a combination of two waves of the same frequency and then verify the result using trigonometrical formulae
P2	solve a practical engineering problem involving an arithmetical progression	M2	use complex numbers to solve a parallel arrangement of impedances giving the answer in both Cartesian and polar form	D2	use numerical integration and integral calculus to analyse the results of a complex engineering problem.
P3	solve a practical engineering problem involving an geometric progression	M3	use differential calculus to find the maximum/minimum for an engineering problem.		
P4	perform the two basic operations of multiplication and division to a complex number in both rectangular and polar form, to demonstrate the different techniques				
P5	calculate the mean, standard deviation and variance for a set of ungrouped data				
P6	calculate the mean, standard deviation and variance for a set of grouped data				

Grading criteria		
To achieve a pass grade the evidence must show that the learner is able to:	**To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:**	**To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:**
P7 sketch the graph of a sinusoidal trigonometrical function and use it to explain and describe amplitude, periodic time and frequency P8 use two of the compound angle formulae and verify their relationship P9 find the differential coefficient for three different functions to demonstrate the use of function of a function and the product and quotient rules P10 use integral calculus to solve two simple engineering problems involving the definite and indefinite integral.		

Essential guidance for tutors

Delivery

Although this unit can be delivered on its own, it requires learners to have successfully completed *Unit 4*: *Mathematics for Technicians* or an equivalent before attempting it. For this reason it should be delivered at a later stage in the course, after a suitable foundation in mathematics and engineering principles has been established.

Every opportunity should be taken to apply and contextualise the underpinning mathematical principles to suit learners' chosen engineering specialism. Tutors could provide a selection of well-prepared, vocationally relevant examples and assignments that are tailored to area-specific programmes of study, as well as selecting specific applications from the suggested option.

Regular opportunities (eg classroom exercises) to address the relevant techniques should be provided as part of formative assessment. Constant feedback, using additional formative tests and coursework that falls outside the formal summative assessment, may be used to aid learning without necessarily being graded.

The unit content does not need to be taught or assessed in order and it is left to centres to decide on their preferred order of delivery.

Note that the use of 'eg' in the content is to give an indication and illustration of the breadth and depth of the area or topic. As such, not all content that follows an 'eg' needs to be taught or assessed.

Assessment

P1 and M1 are probably best assessed through an assignment with learners being given different equations for a meaningful task (eg two operatives producing a certain number of assemblies) for P1.

For M1 learners will need to provide evidence that they can reduce an engineering law (eg gas pressure and temperature $T = \alpha p^y$, current and voltage $I = VR^k$) to straight line form, then use logarithmic graph paper to plot the graph and obtain values for the constants. This is probably best achieved with an assignment in which learners are each given different values.

P2 and P3 can be combined into one assignment, again relevant to an engineering problem (eg the drilling of bore holes for an arithmetic progression solution and the calculation of drill speeds for a geometric progression solution).

For P4 learners could be given different values to demonstrate the two basic operations and this could be linked to M2 to form one assignment.

P5 and P6 could also be linked and assessed by an assignment or short formal test with a relevant application (eg values of resistors, quality control of a product, overtime working).

P7 and P8 could be assessed by a short formal class test. Alternatively an assignment could be used with different values for the graphical output given to different learners. Either approach would help ensure answers are authentic.

P9, P10 and M3 are possibly best assessed as a short exercise or assignment, with learners being given a list of the standard differential coefficients and integrals to use. For P9 each of the questions could be written to assess all the three rules in turn. P10 requires a simple engineering problem (eg indefinite integral given information to find value of constant and hence required equation, definite integral such as area under a curve). M3 is possibly best linked to P9 and P10 with M3 assessing an engineering problem (eg use of differentiation to find the dimensions of a rectangular box to give the maximum volume).

The merit criteria need to build upon the pass criteria, and as such may form an extension to an assessment containing several of the pass criteria as already indicated.

Evidence for the distinction criteria needs to show that learners have a more in-depth knowledge and understanding. Both the distinction criteria could be assessed by a written assignment. For D1 learners firstly need to use a graphical technique to obtain the single wave resulting from a combination of two waves of the same frequency. Each learner could be given a slightly different equation by varying the values of A and B (eg $V_1 = A\sin\left(100\pi t + \frac{2\pi}{5}\right)$, $V_2 = B\sin\left(100\pi t - \frac{2\pi}{9}\right)$). Learners should be encouraged to use a computer package for recording, evaluating and possibly plotting a range of values (eg t from 0 to 0·02s). By using the double angle formulae (eg $Sin\left(A \pm B\right)$) and the expansion of $R\sin(wt \pm \alpha)$ learners could verify their results.

Ideally D2 should provide a comparison between one, two or all three of the numerical integration methods and integration by calculus (eg evaluation of the distance traveled in the first second when a guitar string is plucked about its centre, given an equation for its velocity).

Links to National Occupational Standards, other BTEC units, other BTEC qualifications and other relevant units and qualifications

Unit 4: *Mathematics for Technicians* is an essential prerequisite for this unit and as such must be studied prior to this unit. The unit also links with all other analytically-based mechanical and electrical principles units. It is, therefore, important that these links are reinforced by delivering appropriate specialist units for the learners' own disciplines concurrently.

Learners completing this unit together with *Unit 4*: *Mathematics for Technicians* will be well equipped for progression onto BTEC Higher National Certificate/Diploma courses and first year engineering degree qualifications.

Essential resources

Learners will need to use an electronic scientific calculator and have access to software packages that support the concepts and principles and their application to engineering.

Indicative reading for learners

Bird J — *Engineering Mathematics* (Newnes, 2003) ISBN 0750657766

Tooley M and Dingle L — *BTEC National Engineering* (Newnes, 2002) ISBN 0750651660

Key skills

Achievement of key skills is not a requirement of this qualification but it is encouraged. Suggestions of opportunities for the generation of Level 3 key skill evidence are given here. Tutors should check that learners have produced all the evidence required by part B of the key skills specifications when assessing this evidence. Learners may need to develop additional evidence elsewhere to fully meet the requirements of the key skills specifications.

Application of number Level 3	
When learners are:	**They should be able to develop the following key skills evidence:**
• producing and evaluating solutions to engineering problems using a range of mathematical techniques • producing graphical solutions to problems involving statistical and scientific data.	N3.1 Plan an activity and get relevant information from relevant sources. N3.2 Use this information to carry out multi-stage calculations to do with: a amounts or sizes b scales or proportion c handling statistics d using formulae. N3.3 Interpret the results of your calculations, present your findings and justify your methods.