# Unit 4: Mathematics for Technicians

**NQF Level 3: BTEC National**

**Guided learning hours: 60**

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## Unit abstract

One of the main responsibilities of engineers is to solve problems quickly and effectively. This unit will enable learners to solve mathematical, scientific and associated engineering problems at technician level. It will also act as a basis for progression to study other units both within the qualification such as *Unit 28: Further Mathematics for Technicians* and at BTEC Higher National level.

This unit enables learners to build on knowledge gained at GCSE or BTEC First Diploma level and use it in a more practical context for their chosen discipline. The first learning outcome will develop learners' knowledge and understanding of algebraic methods, from a look at the use of indices in engineering to the use of the algebraic formula for solving quadratic equations. Learning outcome 2 involves the introduction of the radian as another method of angle measurement, the shape of the trigonometric ratios and the use of standard formulae to solve surface areas and volumes of regular solids. Learning outcome 3 requires learners to be able to represent statistical data in a variety of ways and calculate the mean, median and mode. Finally, learning outcome 4 is intended as a basic introduction to the arithmetic of elementary calculus.

This unit acts as an essential basis for successful completion of many of the other units within the qualification.

## Learning outcomes

**On completion of this unit a learner should:**

1 Know how to use algebraic methods

2 Be able to use trigonometric methods and standard formula to determine areas and volumes

3 Be able to use statistical methods to display data

4 Know how to use elementary calculus techniques.

## Unit content

### 1 Know how to use algebraic methods

*Indices and logarithms*: laws of indices ($a^m$ x $a^n$ = $a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$), laws of logarithms (log *A* + log *B* = log *AB*, log $A^n$ = *n* log *A*, log *A* - log *B* = log $\frac{A}{B}$) eg common logarithms (base 10), natural logarithms (base e), exponential growth and decay

*Linear equations and straight line graphs*: linear equations eg *y* = *mx* + *c*; straight line graph (coordinates on a pair of labelled Cartesian axes, positive or negative gradient, intercept, plot of a straight line); experimental data eg Ohm's law, pair of simultaneous linear equations in two unknowns

*Factorisation and quadratics*: multiply expressions in brackets by a number, symbol or by another expression in a bracket; by extraction of a common factor eg *ax* + *ay*, *a*(*x* + 2) + *b*(*x* +2); by grouping eg *ax* — *ay* + *bx* — *by*; quadratic expressions eg $a^2 + 2ab + b^2$; roots of an equation eg quadratic equations with real roots by factorisation, and by the use of formula

### 2 Be able to use trigonometric methods and standard formula to determine areas and volumes

*Circular measure*: radian; degree measure to radians and vice versa; angular rotations (multiples of π radians); problems involving areas and angles measured in radians; length of arc of a circle ($s = r\theta$ ); area of a sector ($A = \frac{1}{2} r^2\theta$)

*Triangular measurement*: functions (sine, cosine and tangent); sine/cosine wave over one complete cycle; graph of tanA as A varies from 0° and 360° (tanA = sin A/cosA); values of the trigonometric ratios for angles between 0° and 360°; periodic properties of the trigonometric functions; the sine and cosine rule; practical problems eg calculation of the phasor sum of two alternating currents, resolution of forces for a vector diagram

*Mensuration*: standard formulae to solve surface areas and volumes of regular solids eg volume of a cylinder = $\pi r^2 h$, total surface area of a cylinder = $2\pi rh + \pi r^2$, volume of sphere = $\frac{4}{3} \pi r^3$, surface area of a sphere = $4 \pi r^2$, volume of a cone = $\frac{1}{3} \pi r^2 h$, curved surface area of cone = $\pi r$ x slant height

### 3 Be able to use statistical methods to display data

*Data handling*: data represented by statistical diagrams eg bar charts, pie charts, frequency distributions, class boundaries and class width, frequency table; variables (discrete and continuous); histogram (continuous and discrete variants); cumulative frequency curves

*Statistical measurement*: arithmetic mean; median; mode; discrete and grouped data

4 **Know how to use elementary calculus techniques**

*Differentiation*: differential coefficient; gradient of a curve $y = f(x)$; rate of change; Leibniz notation ($\frac{dy}{dx}$); differentiation of simple polynomial functions, exponential functions and sinusoidal functions; problems involving evaluation eg gradient at a point

*Integration*: integration as reverse of differentiating basic rules for simple polynomial functions, exponential functions and sinusoidal functions; indefinite integrals; constant of integration; definite integrals; limits; evaluation of simple polynomial functions; area under a curve eg $y = x(x - 3)$, $y = x^2 + x + 4$

## Grading grid

In order to pass this unit, the evidence that the learner presents for assessment needs to demonstrate that they can meet all of the learning outcomes for the unit. The criteria for a pass grade describe the level of achievement required to pass this unit.

| Grading criteria | | |
|---|---|---|
| **To achieve a pass grade the evidence must show that the learner is able to:** | **To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:** | **To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:** |
| P1 manipulate and simplify three algebraic expressions using the laws of indices and two using the laws of logarithms<br>P2 solve a linear equation by plotting a straight-line graph using experimental data and use it to deduce the gradient, intercept and equation of the line<br>P3 factorise by extraction and grouping of a common factor from expressions with two, three and four terms respectively<br>P4 solve circular and triangular measurement problems involving the use of radian, sine, cosine and tangent functions<br>P5 sketch each of the three trigonometric functions over a complete cycle | M1 solve a pair of simultaneous linear equations in two unknowns<br>M2 solve one quadratic equation by factorisation and one by the formula method. | D1 apply graphical methods to the solution of two engineering problems involving exponential growth and decay, analysing the solutions using calculus<br>D2 apply the rules for definite integration to two engineering problems that involve summation. |

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| P6 produce answers to two practical engineering problems involving the sine and cosine rule<br>P7 use standard formulae to find surface areas and volumes of regular solids for three different examples respectively<br>P8 collect data and produce statistical diagrams, histograms and frequency curves<br>P9 determine the mean, median and mode for two statistical problems and explain the relevance of each average as a measure of central tendency<br>P10 apply the basic rules of calculus arithmetic to solve three different types of function by differentiation and two different types of function by integration. | | |

## Essential guidance for tutors

### Delivery

This is a core unit and should be delivered at an early stage of the qualification. It is essential that learners are made aware of the importance of this unit by applying the mathematical concepts to engineering problems whenever possible. It may be beneficial for learners to study *Unit 5: Electrical and Electronic Principles* and/or *Unit 6: Mechanical Principles and Applications* concurrently with this unit. Where this is done, assignments can be designed to cover aspects of more than one unit.

Before starting this unit, learners should be able to demonstrate proficiency in basic mathematical concepts and the use of an electronic scientific calculator to carry out a variety of functions. As a guide to the level required, tutors should consult *Unit 4: Mathematics for Engineering Technicians* in the Edexcel Level 2 BTEC First Certificate and First Diploma in Engineering.

The learning outcomes are ordered logically and could be delivered sequentially. The use of algebraic methods is required before further skills can be developed and used within the unit. Much of the learning outcome 1 can be practised in pure mathematical terms however, tutors could emphasis where these methods would be applied in an engineering context. Obviously much practise in these methods will prove a valuable foundation for the rest of the unit.

Once learners have mastered most of these methods, learning outcome 2 gives opportunities to apply these skills when solving circular and triangular measurement problems. The application of these skills should reflect the context/area of engineering that learners are studying. Formulae do not need to be remembered but correct manipulation of the relevant formulae is very important in solving these problems. Learners should have plenty of practise when drawing graphs for learning outcome 1 and sketching trigonometric functions in learning outcome 2.

During the delivery of this unit there should be opportunities for learners to use statistical data that they have collected from engineering contexts or situations. It is much better to put statistics, required by learning outcome 3, in an engineering context than use generalities such as learners' height etc.

Again, for learning outcome 4 opportunities to practise differentiation and integration must be given to ensure learners understand these activities within the range of the content and before they are given assessment activities. The range of these calculus techniques are listed within the content.

Note that the use of 'eg' in the content is to give an indication and illustration of the breadth and depth of the area or topic. As such, not all content that follows an 'eg' needs to be taught or assessed.

### Assessment

The assessment strategy used will need to cover all the learning outcomes and associated pass criteria but not necessarily all the topics included in the unit content.

Criterion P1 may be best assessed in the form of a short written test and could possibly also include criterion P3.

P2 could be assessed through an assignment using data from either *Unit 5: Electrical and Electronic Principles* and/or *Unit 6: Mechanical Principles and Applications*, which ideally would be delivered concurrently with this unit. If this not possible, learners should be given a range of data sufficient for them to plot the graph and work out the gradient, intercept and the equation. Data forcing them to draw the line of best fit, as opposed to a set of points directly on the graphical line, might be most appropriate.

For P4 learners could be given a range of different values and assessed by an assignment or a short formal test. The problems given should collectively cover radian, sine, cosine and tangent functions. When considering the content part of this learning outcome it is important that these problems give the learner the opportunity to convert multiples of π radians to degrees and vice versa. The circular measurement problems also need to cover the length of an arc and area of a sector as well as areas and angles measured in radians. Obviously the triangular measurement problems are more basic and only expect application of the three functions.

P5 requires learners to sketch each of the three trigonometric ratios and this is probably best done as a classroom exercise. Similarly, P6 could take the form of a written assignment where learners must produce answers to two practical engineering problems involving the sine and cosine rule (eg calculate the phasor sum of two alternating currents and evaluate the resultant and the angle between two forces).

Criterion P7 requires learners to calculate the surface areas and volumes for three different regular solids. This could be achieved through an assignment or perhaps by combining it with other criteria in a short formal test.

An assignment could be used for P8 where learners collect meaningful data (eg classification of workers within their company) and display this information using different graphical methods (eg bar charts). They also need to produce a histogram and plot frequency curves (eg resistance values of 100 resistors or external diameter of pins).

For P9 learners must provide evidence that they are able to determine and then explain the relevance of the mean, median and mode for a set of discrete and grouped data (eg time taken to produce components on a machine rounded to the nearest ten seconds and the 100 resistor values or diameters of pins from P8). This could be done by an assignment. P10 may be assessed through a short formal test, with learners being given a list of the standard differential coefficients and integrals to use.

For M1 learners will need to provide evidence that they can solve a pair of simultaneous linear equations in two unknowns (eg equations formed after the application of Kirchhoff's laws, power transmitted for different belt tensions in a mechanical system). It would be appropriate to use the same assessment method and instrument as P2, possibly combining these two criteria as one assessment activity.

M2 could also be assessed by assignment as it requires learners to evaluate the roots of a quadratic equation by factorisation and by the formula method (eg evaluation of an equation formed after the realisation of a practical situation).

Both the distinction criteria could be assessed through a written assignment. For D1 learners need to apply graphical methods to the solution of two engineering problems involving exponential growth and decay (eg growth of voltage in a capacitor, radioactive decay, application of Taylor's tool life equation $C = VT^n$) and then analyse the results by applying the appropriate method of differential calculus to check the results.

D2 requires learners to demonstrate that they can accurately evaluate two engineering problems involving definite integration (eg area under a velocity-time graph, area under a voltage-current graph).

**Links to National Occupational Standards, other BTEC units, other BTEC qualifications and other relevant units and qualifications**

This unit has strong links with *Unit 5: Electrical and Electronic Principles*, *Unit 6: Mechanical Principles and Applications*, *Unit 11: Further Mechanical Principles and Applications* and *Unit 18: Advanced Mechanical Principles and Applications*. It provides learners with the mathematical basis for all other units within the BTEC National qualification and leads to *Unit 28: Further Mathematics for Technicians* or study at BTEC Higher National Certificate/Diploma level.

**Essential resources**

Learners will need to possess an electronic scientific calculator and have access to software packages that support understanding of the principles and their application to engineering.

**Indicative reading for learners**

**Textbooks**

Bird J — *Engineering Mathematics, Fourth Edition* (Newnes, 2003) ISBN 0750657766

Tooley M and Dingle L — *BTEC National Engineering* (Butterworth-Heinemann, 2002) ISBN 0750651660

## Key skills

Achievement of key skills is not a requirement of this qualification but it is encouraged. Suggestions of opportunities for the generation of Level 3 key skills evidence are given here. Staff should check that learners have produced all the evidence required by part B of the key skills specifications when assessing this evidence. Learners may need to develop additional evidence elsewhere to fully meet the requirements of the key skills specifications.

| **Application of number Level 3** | |
|---|---|
| **When learners are:** | **They should be able to develop the following key skills evidence:** |
| • evaluating solutions to mathematical, scientific and engineering problems<br>• producing graphical solutions to problems involving statistical and scientific data. | N3.1 Plan an activity and get relevant information from relevant sources.<br>N3.2 Use this information to carry out multi-stage calculations to do with:<br>a amounts or sizes<br>b scales or proportion<br>c handling statistics<br>d using formulae.<br>N3.3 Interpret the results of your calculations, present your findings and justify your methods. |